Determination of the $CP\mbox{-violating}$ weak phase γ from the decays $B\to D\pi$

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Abstract. We propose a new method to extract the CP-violating weak phase γ (the phase of V_{ub}^*) in the CKM paradigm of the standard model, using $B^- \to D^0 \pi^- \to f\pi^-$ and $B^- \to \overline{D}^0 \pi^- \to f\pi^-$ decays, where the f's are final states such as $K^+\pi^-$, $K^+\rho^-$, $K\pi\pi$, etc. We also study the experimental feasibility of our new method. With the possibility of new phases in the CKM matrix, we re-examine some of the previously proposed methods to determine γ , and find that it would be in principle possible to identify γ and a new phase angle θ separately.

The source for CP violation in the standard model (SM) with *three* generations is a phase in the Cabibbo– Kobayashi–Maskawa (CKM) matrix [1]. One of the goals of *B* factories is to test the SM through measurements of the unitarity triangle of the CKM matrix. An important way of verifying the CKM paradigm is to measure the three angles [2],

$$\alpha \equiv \operatorname{Arg}[-(V_{td}V_{tb}^*)/(V_{ud}V_{ub}^*)],$$

$$\beta \equiv \operatorname{Arg}[-(V_{cd}V_{cb}^*)/(V_{td}V_{tb}^*)],$$

and

$$\gamma \equiv \operatorname{Arg}[-(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)], \qquad (1)$$

of the unitarity triangle independently of many experimental observables and to check whether the sum of these three angles is equal¹ to 180°, as it should be in the paradigm. The angle β can be determined unambiguously by measuring the time-dependent CP asymmetry in the gold-plated mode $B \rightarrow J/\psi K_S$ [3]. The angle α can be extracted to a reasonable accuracy through the study of CP asymmetry in $B \rightarrow \pi\pi$, combined with the isospin analysis to remove the involved penguin contamination [4], and through the decays $B \rightarrow \rho\pi$ [5] and $B \rightarrow a_0\pi$ [6]. It is well known that among the three angles, γ would be the most difficult to determine in experiment. There have been a lot of works to propose methods for measuring γ using B decays, but at present there is no gold-plated way

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to determine this angle. In particular, a class of methods using $B \rightarrow DK$ decays have been proposed [7–12]. All the methods presented in [7–12] have assumed the *unitarity* of the CKM matrix and, therefore, only one independent phase angle in the CKM matrix.

We present a new method for determining γ which is similar to the Atwood–Dunietz–Soni (ADS) method [9], but we use $B \to D\pi$ decays instead of $B \to DK$ decays used in the ADS method. The CLEO Collaboration have observed [13] that the branching ratio for $B^- \to D^0 \pi^-$ is much larger than that for $B^- \to D^0 K^-$,

$$\frac{\mathcal{B}(B^- \to D^0 K^-)}{\mathcal{B}(B^- \to D^0 \pi^-)} = 0.055 \pm 0.014 \pm 0.005.$$
(2)

We consider the decay processes $B^- \to D^0 \pi^- \to f \pi^-$, $B^- \to \bar{D}^0 \pi^- \to f \pi^-$ and their CP-conjugate processes, where D^0 and \bar{D}^0 decay into common final states $f = K^+\pi^-$, $K^+\rho^-$, $K\pi\pi$, and so forth. We note that the decay mode $B^- \to \bar{D}^0\pi^-$ is severely suppressed relative to the mode $B^- \to D^0\pi^-$, and this fact causes serious experimental difficulties in using $B^- \to \bar{D}^0\pi^-$ decay for the Gronau–London–Wyler (GLW) method [7]. However, in our method one needs not to perform the difficult task of measuring the branching ratio for $B^- \to \bar{D}^0\pi^-$, similar to the case of the ADS method. The detailed experimental feasibility for our new method and the ADS method is given later.

Note that the decay amplitudes of $B^- \to D^0 \pi^-$ and $D^0 \to f$ contain the CKM factors $V_{ud}^* V_{cb}$ and $V_{cd}^* V_{us}$, respectively, while the amplitudes of $B^- \to \bar{D}^0 \pi^-$ and $\bar{D}^0 \to f$ contain the CKM factors $V_{cd}^* V_{ub} = |V_{cd}^* V_{ub}| e^{-i\gamma}$ and $V_{ud}^* V_{cs} = |V_{ud}^* V_{cs}|$, respectively. We define the following quantities: (i = 1, 2)

$$a = A(B^{-} \to D^{0}\pi^{-}) = |A(B^{-} \to D^{0}\pi^{-})|e^{i\delta_{a}},$$

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 $^{^{1}}$ The sum of those three angles, defined as the intersections of three lines, would always be equal to 180° , even though the three lines may not be closed to make a triangle

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$$b = A(B^{-} \to \bar{D}^{0}\pi^{-}) = |A(B^{-} \to \bar{D}^{0}\pi^{-})|e^{-i\gamma}e^{i\delta_{b}},$$

$$c_{i} = A(D^{0} \to f_{i}) = |A(D^{0} \to f_{i})|e^{i\delta_{c_{i}}},$$

$$c'_{i} = A(D^{0} \to \bar{f}_{i}) = |A(D^{0} \to \bar{f}_{i})|e^{i\delta_{c'_{i}}},$$

$$d_{i} = A(B^{-} \to [f_{i}]\pi^{-}),$$
(3)

where A denotes the relevant decay amplitude and the δ 's are the relevant strong rescattering phases. Here $[f_i]$ in d_i means that f_i originates from a D^0 or \overline{D}^0 decay. Similarly, we also define \overline{a} , \overline{b} , \overline{c}_i , \overline{c}'_i and \overline{d}_i as the CP-conjugate decay amplitudes corresponding to a, b, c_i , c'_i and d_i , respectively, such as $\overline{d}_i = A(B^+ \to [\overline{f}_i]\pi^+)$, etc. Note that $|x| = |\overline{x}|$ with $x = a, b, c_i, c'_i$, but in general $|d_i| \neq |\overline{d}_i|$, as shown below. Then, the amplitude d_i can be written as

$$d_{i} = A(B^{-} \rightarrow D^{0}\pi^{-})A(D^{0} \rightarrow f_{i})$$

+ $A(B^{-} \rightarrow \bar{D}^{0}\pi^{-})A(\bar{D}^{0} \rightarrow f_{i})$
= $ac_{i} + b\bar{c}'_{i}$
= $|ac_{i}|e^{i(\delta_{a}+\delta_{c_{i}})} + |b\bar{c}'_{i}|e^{-i\gamma}e^{i(\delta_{b}+\delta_{c'_{i}})}.$ (4)

Thus, $|d_i|^2$ and $|\bar{d}_i|^2$ are given by

$$\begin{aligned} |d_i|^2 &= |ac_i|^2 + |b\bar{c}_i'|^2 + 2|abc_i\bar{c}_i'|\cos(\gamma + \Delta_i), \\ |\bar{d}_i|^2 &= |ac_i|^2 + |b\bar{c}_i'|^2 + 2|abc_i\bar{c}_i'|\cos(\gamma - \Delta_i), \end{aligned}$$
(5)

where $\Delta_i = \delta_a - \delta_b + \delta_{c_i} - \delta_{c'_i}$. We see that $|d_i| \neq |\bar{d}_i|$, unless $\Delta_i = n\pi$ $(n = 0, 1, \cdots)$. The expressions in (5) represent four equations for i = 1, 2. Now let us assume that the quantities $|a|, |c_i|, |c'_i|, |d_i|$ and $|\bar{d}_i|$ are measured by experiment, but |b| is unknown. Then there are the four unknowns $|b|, \gamma, \Delta_1, \Delta_2$ in the above four equations. By solving the equations one can determine γ , as well as the other unknowns such as $|b| = |A(B^- \to \overline{D}^0 \pi^-)|$.

In the ADS method [9], a, b and d_i in (3) are replaced by

$$a = A(B^{-} \to D^{0}K^{-}) = |A(B^{-} \to D^{0}K^{-})|e^{i\delta_{a}},$$

$$b = A(B^{-} \to \bar{D}^{0}K^{-}) = |A(B^{-} \to \bar{D}^{0}K^{-})|e^{-i\gamma}e^{i\delta_{b}},$$

$$d_{i} = A(B^{-} \to [f_{i}]K^{-}).$$
(6)

Then $|d_i|^2$ and $|\bar{d}_i|^2$ can be expressed in the same form as in (5). Therefore, the phase γ can be determined by solving the four equations (for i = 1, 2) with four unknowns $|b|, \gamma$, Δ_1, Δ_2 . The impact on the ADS method due to the large $D^0-\bar{D}^0$ mixing from new physics has been studied in [14].

Now we study the experimental feasibility of our new method and the ADS method, by solving (5) analytically,

$$\cos(\gamma + \Delta_i) = \frac{|d_i|^2 - |ac_i|^2 - |b\bar{c}_i'|^2}{2|ac_i b\bar{c}_i'|},$$

$$\cos(\gamma - \Delta_i) = \frac{|\bar{d}_i|^2 - |ac_i|^2 - |b\bar{c}_i'|^2}{2|ac_i b\bar{c}_i'|}.$$
(7)

To make a rough numerical estimate of the possible statistical error in the determination of γ , we use the experimental result, (2), and the mean values for the CKM elements:

$$\mathcal{B}(B^{-} \to D^{0}\pi^{-}) : \mathcal{B}(B^{-} \to D^{0}K^{-}) : \mathcal{B}(B^{-} \to \bar{D}^{0}K^{-}) : \mathcal{B}(B^{-} \to \bar{D}^{0}\pi^{-}) \simeq |V_{cb}V_{ud}^{*}|^{2} : |V_{cb}V_{us}^{*}|^{2} : |V_{ub}V_{cs}^{*}/N_{c}|^{2} : |V_{ub}V_{cd}^{*}/N_{c}|^{2} \approx A^{2}\lambda^{4} \times (1:\lambda^{2}:\lambda^{2}/36:\lambda^{4}/36) \approx 100:5:0.15:0.007 \sim \mathcal{O}(100): \mathcal{O}(10): \mathcal{O}(0.1): \mathcal{O}(0.01).$$
(8)

where we used $|V_{ub}/V_{cb}| \approx \lambda/2$, the color-suppression factor $N_c = 3$, $\lambda = \sin \theta_C = 0.22$, and $A = V_{cb}/\lambda^2$ is a Wolfenstein parameter. In order to consider the decay parts, c_i , \bar{c}'_i , we choose the modes such as $|\bar{c}'_i| >> |c_i|$, e.g.,

$$\begin{aligned} |c(D^{0} \to K^{+}\pi^{-})|^{2} &: |\bar{c}'(\bar{D}^{0} \to K^{+}\pi^{-})|^{2} \\ &= \mathcal{B}(D^{0} \to K^{+}\pi^{-}) : \mathcal{B}(\bar{D}^{0} \to K^{+}\pi^{-}) \\ &= (1.5 \pm 0.3) \times 10^{-4} : (3.8 \pm 0.1) \times 10^{-2} \\ &\sim \mathcal{O}(1) : \mathcal{O}(100), \end{aligned}$$
(9)

which yields

$$|ac_i|^2(\pi) : |ac_i|^2(K) : |b\bar{c}_i'|^2(K) : |b\bar{c}_i'|^2(\pi) \propto \mathcal{O}(100) : \mathcal{O}(10) : \mathcal{O}(10) : \mathcal{O}(1).$$
(10)

Therefore, if we assume the 1% level precision in the experimental determination for a product of branching ratios², e.g., $\Delta[\mathcal{B}(B^- \to D^0\pi^-) \times \mathcal{B}(D^0 \to K^+\pi^-)] = 1\%$, then we can set the numerical values, for $B^{\pm} \to D(\to f_i)\pi^{\pm}$,

$$|ac_i|^2(\pi) \approx 100 \pm 1, \quad |b\bar{c}'_i|^2(\pi) \approx 1 \pm 0.1,$$
 (11)

and for $B^{\pm} \to D(\to f_i)K^{\pm}$,

$$|ac_i|^2(K) \approx 10 \pm 0.3, \quad |bc_i'|^2(K) \approx 10 \pm 0.3.$$
 (12)

Then we can make a rough estimate for the statistical error from (7):

$$\Delta [\cos(\gamma + \theta \pm \Delta_i)(B^{\pm} \to D(\to f_i)\pi^{\pm})] \sim 0.1,$$

$$\Delta [\cos(\gamma \pm \Delta_i)(B^{\pm} \to D(\to f_i)K^{\pm})] \sim 0.05.$$
(13)

We find that the ADS method can give approximately a twice better precision statistically for the determination of γ , compared to our new method.

In fact, there is a general theorem [16]:

$$N_B \propto 1/(\mathcal{B}(B \to f)A_f^2),$$
 (14)

where N_B is the number of B mesons needed, \mathcal{B} the branching ratio of a decay mode, $B \to f$, and A_f the relevant asymmetry. Now, as shown in (2), $\mathcal{B}(\text{ADS method})/$

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² At present, the experimental data for this product of branching ratio is given in about 29% level precision, using a few times $10^6 B$ mesons [15]. Thus, to obtain the data in 1% level precision, one needs to have about $10^9 B$'s, which can be achieved at hadronic B experiments such as BTeV and LHC-B, where more than $10^{10} B$ mesons will be produced per year

 $\mathcal{B}(\text{our method}) \simeq 0.05$. To determine the relevant asymmetry A_f , one has to calculate the following:

$$A_{f} = \frac{|d_{i}|^{2} - |d_{i}|^{2}}{|d_{i}|^{2} + |\bar{d}_{i}|^{2}}$$

$$= \frac{-2|abc_{i}\bar{c}'_{i}|\sin\gamma\sin\Delta_{i}}{|ac_{i}|^{2} + |b\bar{c}'_{i}|^{2} + 2|abc_{i}\bar{c}'_{i}|\cos\gamma\cos\Delta_{i}}$$

$$\sim \frac{2|abc_{i}\bar{c}'_{i}|}{|ac_{i}|^{2} + |b\bar{c}'_{i}|^{2}}.$$
(15)

For simplicity, we have considered the maximum asymmetry in both methods. Using the experimental values given in (2), (8)–(12), we can easily get A_f (ADS method)/ A_f (our method) \simeq (5–10). Therefore, N_B (ADS method)/ N_B (our method) \sim (1–0.2), which is exactly consistent with the above prediction, (13), where we have predicted the possible precision with the same number of B mesons.

We note that our new method may have other advantages:

- (1) The values of $|d_i|^2$ and $|\bar{d}_i|^2 \propto \mathcal{B}(B^{\pm} \to [f_i]\pi^{\pm})$ are an order of magnitude larger than $|d_i|^2$ and $|\bar{d}_i|^2 \propto \mathcal{B}(B^{\pm} \to [f_i]K^{\pm})$. Therefore, if the present asymmetric *B* factories of Belle and Babar can produce only a handful of such events because of the limited detector and trigger efficiencies, our new method may be the first measurable option.
- (2) Systematic errors could be much smaller for our new method due to the final state particle identification, i.e. a fewer number of the final state pions due to $K \to \pi\pi$, and the reconstruction of K.

Now we would like to make comments on new physics effects in the determination of the weak phase γ . There can be two independent approaches to find out new physics beyond the SM, if it exists.

- (1) The unitarity of the CKM matrix can be assumed. In this case, new physics effects can only come out from new virtual particles or through new interactions in penguin or box diagrams in B meson decays. If this is the case, all the methods which we mentioned above will extract exactly the same γ .
- (2) The CKM matrix can be generalized to the nonunitary matrix. In this case, new physics effects can appear even in tree diagram decays. And the values of γ extracted from each method can be different. Therefore, we will describe in more detail this second case.

In fact, in models beyond the SM, the CKM matrix may not be unitary; for instance, in a model with an extra down quark singlet (or more than one), or an extra up quark singlet, or both up and down quark singlets, the CKM matrix is no longer unitary [17,18]. If the unitarity constraint of the CKM matrix is removed, the generalized CKM matrix possesses 13 independent parameters (after absorbing 5 phases into quark fields) – it consists of 9 real parameters and 4 independent phase angles. The generalized CKM matrix can be parameterized by [19]

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{i\delta_{13}} \\ |V_{cd}| & |V_{cs}|e^{i\delta_{22}} & |V_{cb}| \\ |V_{td}|e^{i\delta_{31}} & |V_{ts}| & |V_{tb}|e^{i\delta_{33}} \end{pmatrix}.$$
 (16)



Fig. 1. Three triangles constructed from various $B \to DK$ decays [11]. With a new phase in the CKM matrix, one would extract ($\gamma + \theta$), rather than γ , using the method in [11], as shown in the figure

With the possibility of the non-unitary CKM matrix, one has to carefully examine the effects from the non-unitarity on the previously proposed methods where the unitarity of the CKM matrix was assumed to test the SM for CPviolation. From now on, we set $\gamma \equiv -\delta_{13}$ and $\theta \equiv -\delta_{22}$.

In the parameterization given in (16), using our method, c'_i in (3) is replaced by

$$c'_{i} = |A(D^{0} \to \bar{f}_{i})| e^{\mathrm{i}\theta} e^{\mathrm{i}\delta_{c'_{i}}}.$$
(17)

This leads to the result that the phase γ in the expressions for $|d_i|^2$ and $|\bar{d}_i|^2$ in (5) should be replaced by $(\gamma + \theta)$. Therefore, in this case, our method can measure the nonunitary phase $(\gamma + \theta)$.

In the ADS method, besides that c'_i is changed into the one in (17), the phase γ in b is also replaced by $(\gamma - \theta)$. As a result, the new phase θ is automatically cancelled and disappears in the expressions for $|d_i|^2$ and $|\bar{d}_i|^2$. Thus, the ADS method would still measure γ which is the phase of V_{ub}^{*} .

 V_{ub}^* . The GLW method [7] was suggested for extracting γ from measurements of the branching ratios of decays $B^{\pm} \to D^0 K^{\pm}, B^{\pm} \to \overline{D}^0 K^{\pm}$ and $B^{\pm} \to D_{CP} K^{\pm}$, where D_{CP} is a CP eigenstate. However, the GLW method suffers from serious experimental difficulties, mainly because the process $B^- \to \bar{D}^0 K^-$ (and its *CP*-conjugate process $B^+ \to D^0 K^+$) is difficult to measure in experiment. That is, the rate for the CKM- and color-suppressed process $B^- \to \bar{D}^0 K^-$ is suppressed by about two orders of magnitudes relative to that for the CKM- and color-allowed process $B^- \to D^0 K^-$, and it causes experimental difficulties in identifying \bar{D}^0 through $\bar{D}^0 \to \bar{K}^+ \pi^-$ since doubly Cabibbo-suppressed $D^0 \to K^+\pi^-$ following $B^- \to D^0 K^$ strongly interferes with $\bar{D}^0 \to K^+\pi^-$ following the rare process $B^- \to \overline{D}{}^0 K^-$. With the non-unitary CKM matrix, this method would measure the angle $(\gamma - \theta)$ (see Fig. 1), instead of γ as originally proposed in [7].

In [11] two groups, Gronau and Rosner (GR), and Jang and Ko (JK), propose a method to extract γ by exploiting Cabibbo-allowed decays $B \to DK^{(*)}$ and using the isospin relations. In the GR/JK method, the decay modes $B \to DK$ with the quark process $b \to u\bar{c}s$ contain the CKM factor $|V_{ub}V_{cs}^*|e^{-i(\gamma-\theta)}$ and their amplitudes can be written as

$$A(B^{-} \to \bar{D}^{0}K^{-}) = \left(\frac{1}{2}A_{1}e^{i\delta_{1}} + \frac{1}{2}A_{0}e^{i\delta_{0}}\right)e^{-i(\gamma-\theta)},$$

$$A(B^{-} \to D^{-}\bar{K}^{0}) = \left(\frac{1}{2}A_{1}e^{i\delta_{1}} - \frac{1}{2}A_{0}e^{i\delta_{0}}\right)e^{-i(\gamma-\theta)},$$

$$A(\bar{B}^{0} \to \bar{D}^{0}\bar{K}^{0}) = A_{1}e^{i\delta_{1}}e^{-i(\gamma-\theta)},$$

(18)

where A_i and δ_i denote the amplitude and the strong rescattering phase for the isospin i state. Note that the weak phase angle $(\gamma - \theta)$ appears in (18), rather than γ , as in [11]. In this method, three triangles are drawn to extract 2γ , using the isospin relation

$$A(B^{-} \to \bar{D}^{0}K^{-}) + A(B^{-} \to D^{-}\bar{K}^{0}) = A(\bar{B}^{0} \to \bar{D}^{0}\bar{K}^{0})$$
(19)

and the following relations:

$$A(B^{-} \to D_{1}K^{-}) = A(\bar{B}^{0} \to D_{1}\bar{K}^{0}) + \frac{1}{\sqrt{2}}A(\bar{B}^{0} \to D^{+}K^{-}),$$

$$A(B^{+} \to D_{1}K^{+}) = A(B^{0} \to D_{1}K^{0}) + \frac{1}{\sqrt{2}}A(B^{0} \to D^{-}K^{+}), \quad (20)$$

where D_1 is a CP eigenstate of the D meson, defined by

$$D_1 = \frac{1}{\sqrt{2}} (D^0 + \bar{D}^0).$$

The appearance of $(\gamma - \theta)$ in (18) results in extraction of $2(\gamma - \theta)$, by this method, rather than 2γ as in the above reference, as shown in Fig. 1.

In Fig. 1, $A(B^- \to D_1 K^-)$, $A(B^- \to \overline{D}{}^0 K^-)$ and the thick solid line form a triangle, and $A(B^+ \rightarrow D_1 K^+)$, $A(B^+ \to D^0 K^+)$ and the other thick solid line form another triangle. These two triangles are those used in the GLW method and the thick solid lines correspond to

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$$\frac{1}{\sqrt{2}}A(B^- \to \bar{D}^0 K^-)$$
$$\frac{1}{\sqrt{2}}A(B^+ \to D^0 K^+),$$

respectively, which are very difficult to measure in experi-
ment. Obviously, assuming that the GLW method works,
the method would extract
$$2(\gamma - \theta)$$
 rather than 2γ . We
note that the impact on the GLW method from the siz-
able $D^0 - \overline{D}^0$ mixing via new physics effects has been also
investigated in [20].

In conclusion, we have presented a new method to determine $\gamma = \operatorname{Arg}(V_{ub}^*)$ in the CKM paradigm of the SM, using $B^- \to D^0 \pi^- \to f \pi^-$ and $B^- \to D^0 \pi^- \to f \pi^-$

decays, where $f = K^+\pi^-$, $K^+\rho^-$, $K\pi\pi$, etc. The experimental feasibility of our method, comparing with the ADS method, has been studied. Although experimentally challenging at the present asymmetric B factories of Belle and Babar, the analysis using our method can be carried out in detail in hadronic B experiments such as BTeV and LHC-B, where more than 10^{10} B mesons will be produced per year.

With the possibility of new phases in the CKM matrix, we have re-examined some of the previously proposed methods for determining the weak phase γ using $B \to DK$ or $B \to D\pi$ decays. We have shown that our method would extract $(\gamma + \theta)$ with the new phase θ . The ADS method would measure γ , while the GLW method or the GR/JK method would measure $(\gamma - \theta)$. Thus, if one uses the above methods independently and compares the results, it would be in principle possible to identify γ and θ separately. If this is the case and θ is not negligible, this would be a clear indication of the new phase in the CKM matrix, i.e. an effect from new physics. We note that, in fact, inconsistencies between the values of γ can arise from final state interactions (FSIs) which are known to be important in K decays and, consequently, may modify a quark-level description of B decays. Therefore, there is a possibility that FSIs may cause a problem in identifying the new phase θ by using various methods mentioned above.

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